LEVEL CROSSING PROPERTIES OF ECO-HYDROLOGICAL SYSTEMS DRIVEN BY DICHOTOMIC MARKOV NOISE

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1. Introduction

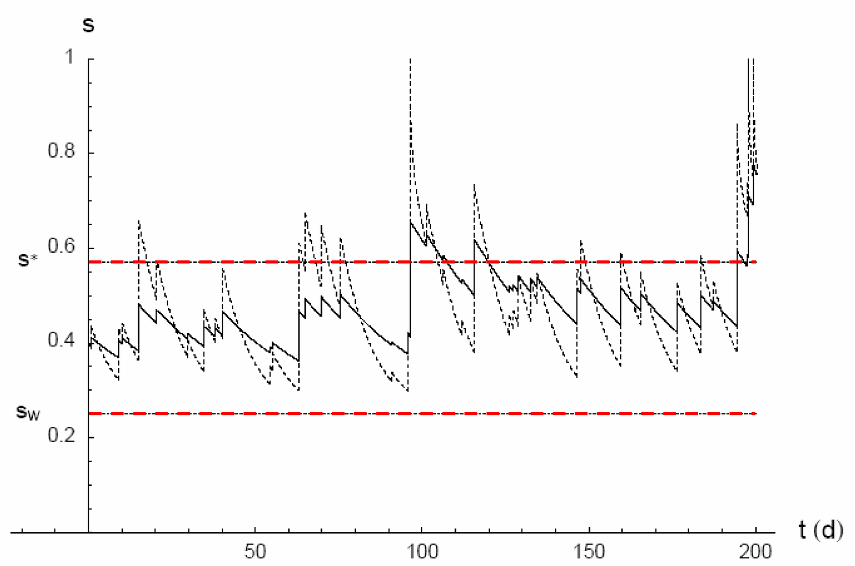
Reduction of soil moisture content during droughts triggers a sequence of damages in plants, called *water stress*. Vegetation starts to suffer water stress at the soil moisture level corresponding to incipient stomatal closure, s^*. Since soil moisture dynamics are stochastic, the duration below s^* is a random variable. The average value of this duration was derived analytically for the stochastic model of soil moisture dynamics treated by *Laio et al. [Adv. Water Resour. 24 725-744, (2001)]*. Duration and frequency of excursions below s^*, and average intensity of deficits may be combined to determine a measure of plant water stress

2. Stochastic soil moisture dynamics

Equation (1) describes the soil moisture balance at a point [*Laio et al. (2001)*],

$$nZ_r \frac{ds(t)}{dt} = \varphi[s(t);t] - \chi[s(t)], \qquad (1)$$

where n is soil porosity, Z_r is the depth of active soil, $s(t)$ is the relative soil moisture content ($0 \le s(t) \le 1$), t is time, $\varphi[s(t);t]$ is the rate of infiltration from stochastic rainfall, $\chi[s(t)]$ is the rate of soil moisture losses from the active soil (evaporation, transpiration, leakage). The terms in Eq. (1) can be modeled in a relatively simple way, in order to obtain the general solution for the probability density function of soil moisture ***p(s)*** (Fig. 1)



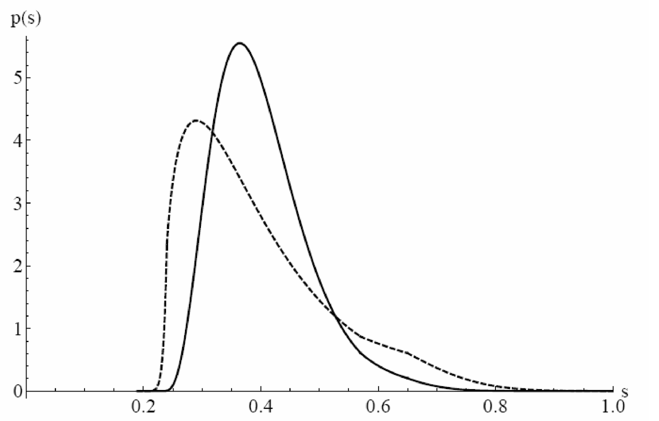
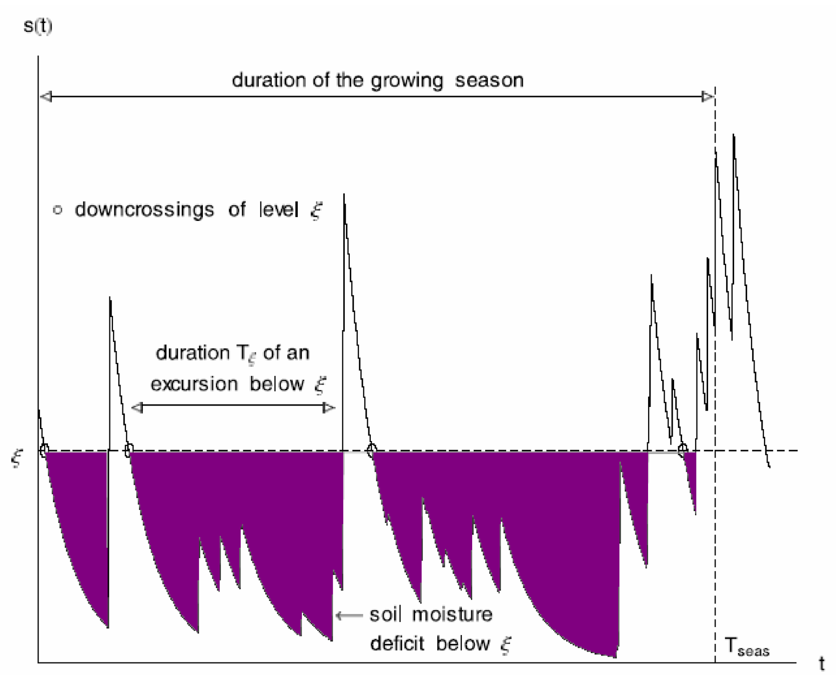


Fig.1 Example of traces of soil moisture and corresponding pdfs for two different values of Z_r

3. Crossing properties of soil moisture levels

The stochastic description of soil moisture leads to the definition of two fundamental variables: 1) the mean length $T_{\xi DOWN}$ of time intervals in which soil moisture is below a threshold ξ [Eq. (2)]; and 2) the mean length $T_{\xi UP}$ of time intervals in which soil moisture is above ξ [Eq. (3)]



$$T_{\xi\,DOWN} = \frac{P(\xi)}{\rho(\xi)\,p(\xi)}, \qquad (2)$$

$$T_{\xi\,UP} = \frac{1-P(\xi)}{\rho(\xi)\,p(\xi)}, \qquad (3)$$

where $P(\cdot)$ is the cumulative density function, and $\rho(\cdot)$ is the loss function describing soil moisture losses

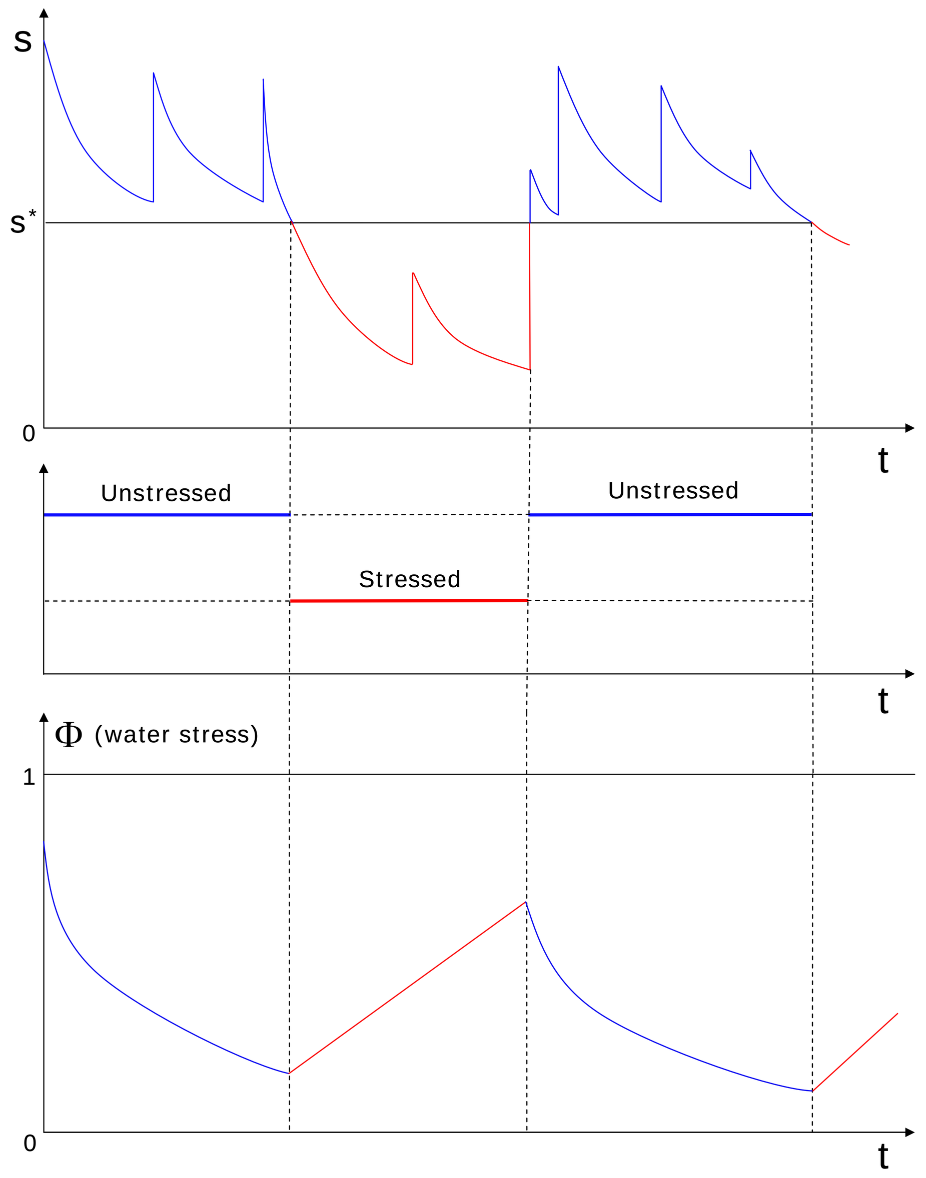
Fig. 2 Temporal evolution of a soil moisture process and its crossing properties with respect to a threshold ξ

When soil moisture is below the level of incipient stomatal closure, s^*, vegetation starts to suffer water stress. Hence, evaluation of Eq. (2) and Eq. (3) for $\xi = s^*$ provides the crossing properties of vegetation water stress [Eqs. (4) and (5)]

$$T_{s^*DOWN} = \frac{P(s^*)}{\rho(s^*)\,p(s^*)} \qquad (4)$$

$$T_{s^*UP} = \frac{1-P(s^*)}{\rho(s^*)\,p(s^*)} \qquad (5)$$

4. Vegetation water stress as a dichotomic Markov process



The temporal evolution of water stress can be described as an alternation between two different states (***stressed*** and ***unstressed***)

From the crossing properties of soil moisture [Eqs. (4) and (5)] it is possible to define the transition probabilities, k_1 and k_2, between the two states of the dichotomic process [Eq. (6)]

$$k_1 = \frac{1}{T_{s^*DOWN}}, \quad k_2 = \frac{1}{T_{s^*UP}}. \qquad (6)$$

During the unstressed state the *dynamic cumulated stress* of vegetation, Φ, is reduced; during the stressed state Φ increases (Fig. 3)

Fig. 3 Schematization of the dynamic process leading to the analysis of vegetation dynamic stress

The temporal evolution of dynamic stress is described as a dichotomic Markov process with transition probabilities k_1 and k_2

$$\frac{d\Phi}{dt} = \begin{cases} \beta & \text{if } s<s^* \quad (7a) \\ -\gamma\Phi & \text{if } s>s^* \quad (7b) \end{cases}$$

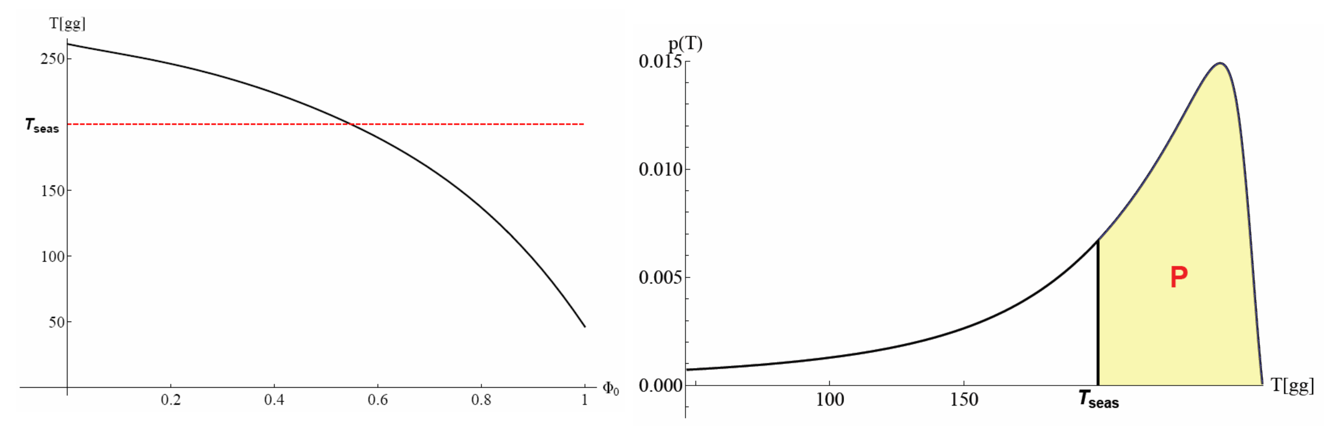
- β is a measure of the ***resistance*** of vegetation
- γ is a measure of the ***resilience*** of vegetation

Other expressions for dynamic stress can be used. For the sake of simplicity, we use two simple functions in order to obtain qualitative results

5. Evaluation of mean lifetime of vegetation

The theory of ***mean first-passage times*** (MFPT) of processes driven by Markovian noise is used to obtain analytical expressions for the conditions of water stress [*Balakrishnan et al. Phys. Rew. A 38(4) 4213-4222, (2001)*]. In particular, we compute the time, T, it takes for vegetation to die (i.e., $\Phi=1$) provided that the initial value of stress is Φ_0 (Fig. 4)

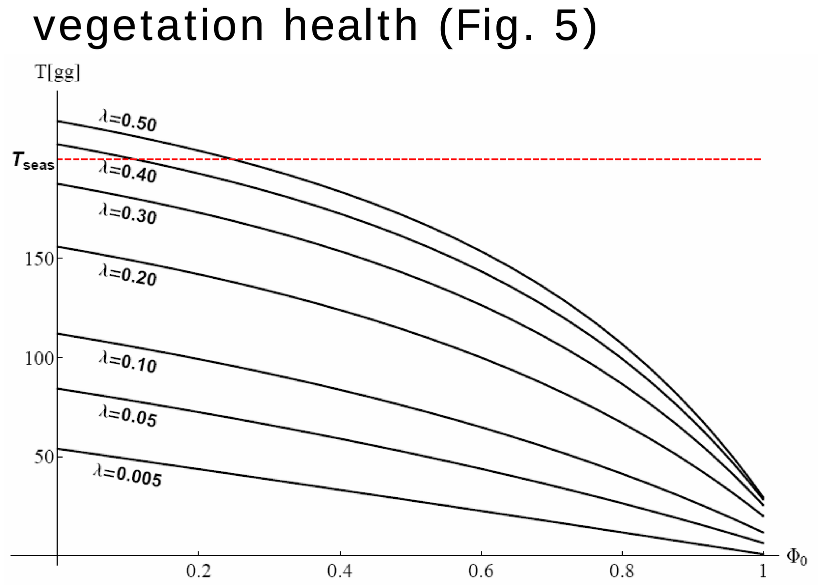
Fig. 4 Relation between mean vegetation lifetime, T, and initial stress Φ_0, and probability distribution of mean vegetation lifetime



Other outcomes are the steady state pdf of T, and the probability of survival to dry conditions, P, i.e. of having a survival time longer than a growing season ($T > T_{seas}$)

The influence of different parameters on mean lifetime of vegetation in arid and semiarid ecosystems is investigated. The main parameters are the total rainfall $\Theta = T_{seas}\alpha\lambda$, the soil type, and the depth of active soil Z_r

Given a constant total rainfall during the growing season, $\Theta = T_{seas}\alpha\lambda$, it is possible to observe an optimal combination between α and λ for vegetation health (Fig. 5)



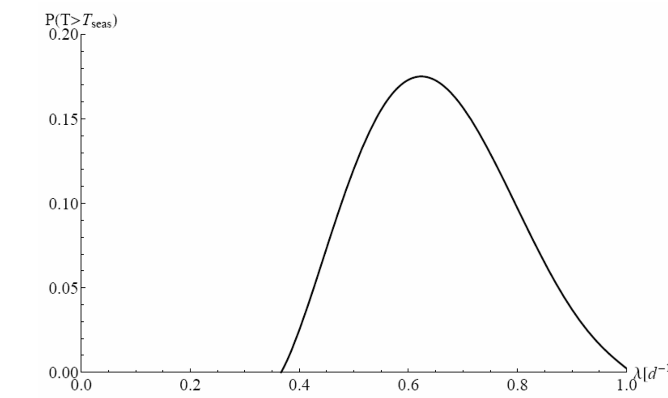
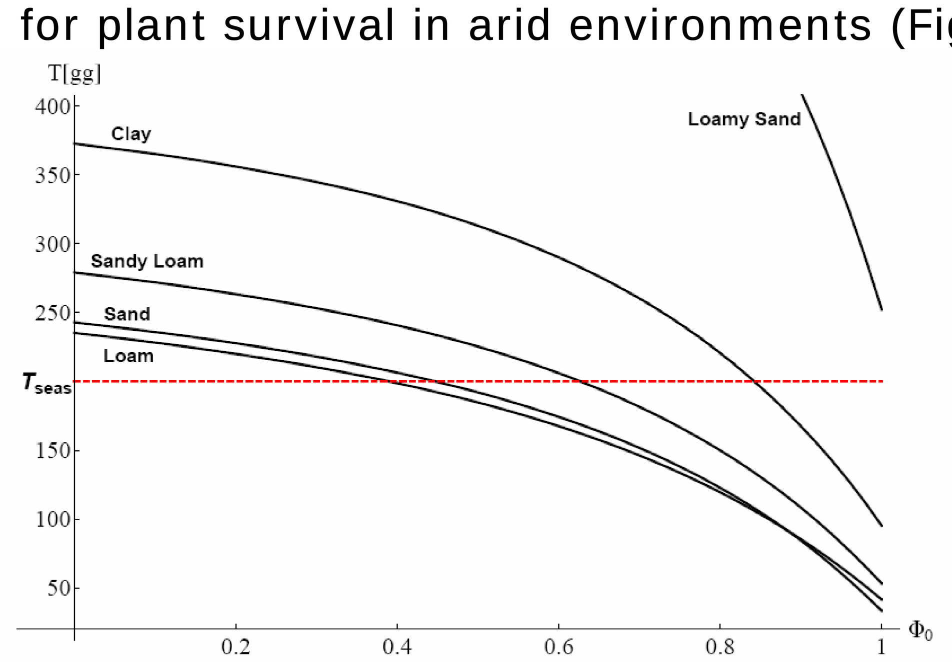


Fig. 5 Dependence of lifetime, T, and probability of survival, P, for constant total rainfall Θ=730mm (β=0.02, γ=0.1, Z_r=30 cm)

The mean interval between rainfall events, $1/\lambda$, seems to play the main role: when λ tends to 0 or to 1, the mean lifetime falls to very small values

When dealing with soil type, loamy sand seems to be the most favourable for plant survival in arid environments (Fig. 6)



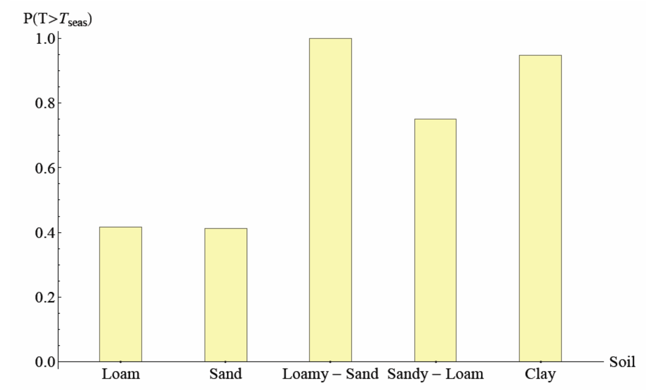
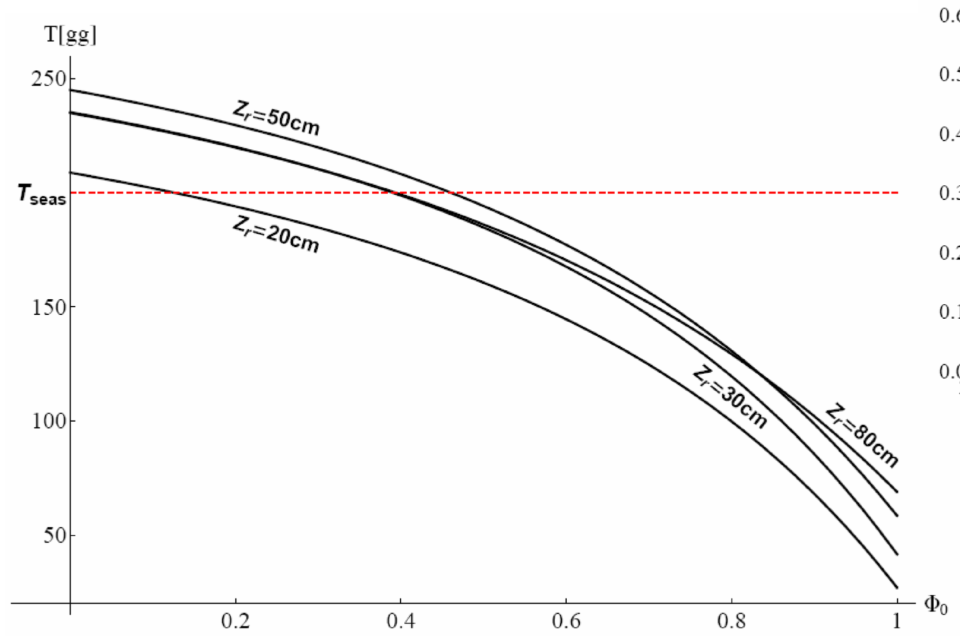


Fig. 6 Dependence of lifetime, T, and probability, P, of having a lifetime longer than the growing season for different soils (β=0.02, γ=0.1, Z_r=30 cm, λ=0.21 d^{-1}; α=2 cm)

The analyses show a negligible influence of active soil depth in the dynamics of water stress. Only with shallow active soils the mean lifetime is substantially reduced (Fig. 7)



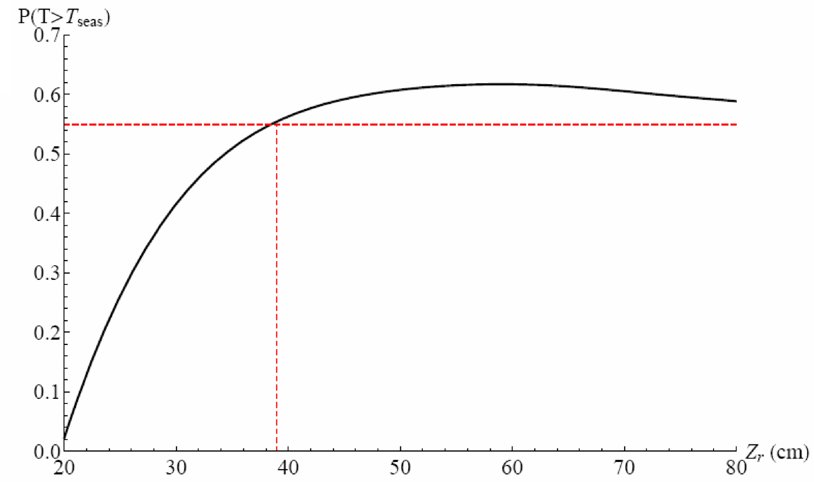


Fig. 7 Dependence of lifetime, T, and probability of survival, P, for different depths of active soil (β=0.02, γ=0.1, λ=0.21 d^{-1}; α=2 cm, loamy soil)

6. Conclusions

A new approach is used to estimate vegetation water stress properties. The temporal evolution of water stress is described as a dichotomous Markov process, i.e. a process with non-null autocorrelation function. Hence, the theory of mean first-passage times (MFPT) of processes driven by Markovian noise can be used to find analytical expressions describing the persistence of water stress dynamics and the ability of vegetation to survive in arid and semi-arid ecosystems