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Representation of space–time variability of soil moisture

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A simplified spatial-temporal soil moisture model driven by stochastic spatial rainfall forcing is proposed. The model is mathematically tractable, and allows the spatial and temporal structure of soil moisture fields, induced by the spatial-temporal variability of rainfall and the spatial variability of vegetation, to be explored analytically. The influence of the main model parameters, reflecting the spatial scale of rain cells, the soil storage capacity, the rainfall interception and the soil water loss rate (representing evaporation and deep infiltration) is investigated. The variabilities of the spatially averaged soil moisture process, and that averaged in both space and time, are derived. The present analysis focuses on spatially uniform vegetation conditions; a follow-up paper will incorporate stochastically heterogeneous vegetation.

Keywords: poisson process; soil moisture; spatial-temporal correlation; stochastic rainfall model

1. Introduction

Soil moisture constitutes the physical linkage between soil, climate and vegetation. Its relevance has been underlined by numerous authors in the last few years (e.g. Pan *et al.* 2003; Albertson & Montaldo 2003; Rodríguez-Iturbe & Porporato 2004) and its dynamic representation remains a central issue for many different research fields. Soil moisture variability in space and time impacts many different processes acting at various scales ranging from point, hillslope, basin, up to global scale. (i) At the point scale, for instance, soil moisture is crucial for the infiltration process (e.g. Raats 2001), plant dynamics and the biogeochemical cycles (e.g. Porporato *et al.* 2001; Porporato *et al.* 2003). (ii) Its distribution across the hillslope represents a controlling factor in the hydrological and

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geotechnical processes responsible for slope instability and land slides (e.g. Montgomery & Dietrich 1994). (iii) Water resources management is focused on drought assessment (e.g. Palmer 1965), flood prediction and water-balance that are strongly influenced by the basin state in terms of the average soil moisture condition (e.g. Manfreda *et al.* 2005). (iv) At the large-scale (e.g. regional or continental), soil moisture and atmospheric phenomena are mutually interacting (e.g. Entekhabi *et al.* 1996).

Space–time variability of soil moisture is a challenging topic for hydrologists. Recent research has received significant input through the experimental campaigns carried out by using extended measurements through portable time domain reflectometry TDR (e.g. Tarrawarra and Mahurangi experiments) and active or passive microwave sensors (e.g. Monsoon '90; Washita '92, '94 and SGP '97, '99). These experiments have increased our understanding of the temporal variability and of the spatial structure of the soil moisture fields (e.g. Rodríguez-Iturbe *et al.* 1995).

Entekhabi & Rodríguez-Iturbe (1994) proposed a description of the space–time soil dynamics based on a stochastic differential equation where the soil was interpreted as a finite bucket, the rainfall as spatial stochastic forcing, the losses (evapotranspiration and runoff) were considered a linear function of the soil moisture, and runoff spatial dynamics were represented by a diffusive term. The analysis was made in the frequency domain, through the hydrological gain function relating the power spectrum of rainfall intensity to that of soil moisture.

The present research presents a renewed attempt to study the dynamics of soil moisture in the spatial and temporal domain through the use of a stationary random process, which is based on a simple water balance model in which soil moisture increases due to precipitation, and decreases due to evapotranspiration and leakage. The rainfall representation (§2) assumes that rain cells occur in a stochastic Poisson process in space and time. Each cell is circular with a random spatial extent. Two variants of the model are considered. In the first (M_1), the total (random) rain depth occurs instantaneously, while in the second (M_2, proposed and investigated in a more general context by Cox & Isham (1988)) the cells have a random intensity for a random duration. The former is a reasonable approximation as long as soil moisture is considered only at daily (or longer) time-scales and is the rainfall model used by the authors in previous work on soil moisture dynamics (e.g. Rodríguez-Iturbe *et al.* 1999; Laio *et al.* 2001; Porporato *et al.* 2004). As will be seen, under suitable conditions, variant M_2 tends to M_1 and the results are similar in regard to the statistical characterization of the soil moisture.

The advantage of these rainfall models is that they allow a quantitative description of the spatial and temporal variability of relative soil moisture fields (§3). Results are presented (§4) for soil moisture properties both at a single space–time point, and also for the field averaged over space and/or time, the latter being useful for comparison with observed data.

Spatial dependence in soil moisture processes results not only because the rainfall input is spatially correlated, but also due to the effects of correlated vegetation cover and soil properties, as well as surface topography. In the present paper, we concentrate on the spatial soil moisture dynamics induced by spatial variability in rainfall and, as in Rodríguez-Iturbe *et al.* (1999) and Laio *et al.* (2001), assume a homogenous soil and vegetation, together with a flat landscape. Runoff and lateral soil moisture redistribution are assumed to be negligible components of water balance. We consider vertically averaged soil moisture content and, even when a temporally distributed rainfall model is assumed, the dynamics will be interpreted at a daily time-scale so that no effects of diurnal fluctuations in temperature on evapotranspiration need to be modelled. Similarly, we consider the dynamics within a single season, and in not very large spatial scales so that neither seasonality nor feedback between soil moisture and rainfall need to be included. The case of heterogeneous soil and vegetation is discussed in a forthcoming paper (Rodríguez-Iturbe *et al.* submitted).

2. Space–time rainfall models

Rainfall occurrences are modelled by a sequence of circular rain cells that occur in a Poisson process of rate λ in space and time. Each cell is characterized by a random radius, W. In the first variant of our rainfall model, M_1, rainfall is instantaneous in time and constant over the spatial extent of the cell, with a random total depth Y (mean μ_Y).

The variables W and Y are assumed independent, and the pairs (W, Y) are independent and identically distributed over the cells. It follows that at a fixed location, say A, rain events occur in a temporal Poisson process of rate $\lambda' = \lambda \mathrm{E}(\pi W^2)$, while events occur simultaneously at locations A and B, l apart, in a temporal Poisson process of rate $\lambda''(l) = \lambda \mathrm{E}[W^2 C(l/W)]$, where

$$C(u) = \begin{cases} 2\cos^{-1}(u/2) - u(1-u^2/4)^{1/2}, & 0 \le u \le 2, \\ 0, & 2 \le u, \end{cases} \tag{2.1}$$

is the area of overlap of two unit discs, with centres a distance u apart.

For analytical purposes, it is sometimes convenient to approximate the overlap function C defined in equation (2.1) as (see Cox & Isham 1988)

$$C_k(u) = \begin{cases} \pi - ku, & 0 \le u \le 2, \\ 0, & 2 \le u, \end{cases} \tag{2.2}$$

for a suitable constant k.

If the cell radii, W, are exponentially distributed with parameter $\rho\,(\rho = \mu_W^{-1})$, then $\lambda' = (2\pi\lambda)/\rho^2$. In this case, approximating $C(u)$ by $C_k(u)$ as in equation (2.2) gives

$$\lambda''(l) \simeq \lambda \left[\frac{2\pi}{\rho^2} + (\pi - k)\frac{l}{\rho} + \left(\frac{\pi}{4} - \frac{k}{2}\right) l^2 \right] \mathrm{e}^{-\rho l/2}.$$

It is often convenient to assume $k = \pi/2$, although this value tends to give a slight overestimate of $C(u)$. In this case, we obtain

$$\lambda''(l) \simeq \frac{2\pi\lambda}{\rho^2}\left(1 + \frac{\rho l}{4}\right)\mathrm{e}^{-\rho l/2}. \tag{2.3}$$

In an alternative variant of the rainfall model, M_2, the rain cells occur as above, but each cell has a random duration, D and intensity, X, so that the total depth of the cell is XD. The variables W, D and X are mutually independent and the triples (W, D, X) are independent and identically distributed over the cells. In this model, the cells overlap temporally as well as spatially so that, at any particular spatial location $\boldsymbol{u}$ and time t, the rainfall process is the superposition of all cells overlapping the space–time point $(\boldsymbol{u},t)$. Thus, the total intensity $\tilde{Y}(u, t)$ can be expressed as

$$\tilde{Y}(\boldsymbol{u}, t) = \int_{\tau=-\infty}^{t} \int_{w\in\mathbb{R}^2} I(\boldsymbol{w}, \tau; \boldsymbol{u}, t) X(\boldsymbol{w}, \tau) \mathrm{d}N(\boldsymbol{w}, \tau), \tag{2.4}$$

where N is the counting measure of the Poisson process of cell origins, $X(w, \tau)$ is the intensity of the cell with origin $(\boldsymbol{w}, \tau)$ and $I(\boldsymbol{w}, \tau; \boldsymbol{u}, t)$ is an indicator variable taking the value 1 if, at time t, that cell is still alive and covers $\boldsymbol{u}$, and taking the value 0 otherwise. Note that $\tilde{Y}$ denotes the total *intensity* of the rainfall process in model variant M_2, while Y denotes the *depth* of an instantaneous rain event in variant M_1.

Properties of model M_2 can be found in Cox & Isham (1988), who discuss a more general model in which the cells have non-zero velocities. If the rainfall intensity at an arbitrary location, A, is denoted by $\tilde{Y}_{\mathrm{A}}$, it follows that the mean intensity is given by

$$\mathrm{E}(\tilde{Y}_{\mathrm{A}}(t)) = \lambda' \mu_D \mu_X, \tag{2.5}$$

where μ_D is the mean storm duration and μ_X is the mean cell intensity. If the cell durations are exponentially distributed with parameter η, the covariance of the rainfall intensities at locations A and B, l apart, and with a temporal lag $h \geq 0$, is given by

$$\mathrm{cov}(\tilde{Y}_{\mathrm{A}}(t), \tilde{Y}_{\mathrm{B}}(t+h)) = \lambda''(l) \mathrm{E}(X^2) \frac{\mathrm{e}^{-\eta h}}{\eta}. \tag{2.6}$$

Letting $l \to 0$ and $h \to 0$ in (2.6), so that the locations A and B coincide and $\lambda''(l) \to \lambda'$, it follows that the variance of the rainfall process has the form

$$\sigma_{\tilde{Y}}^2 = \mathrm{var}(\tilde{Y}_{\mathrm{A}}(t)) = \frac{\lambda' \mathrm{E}(X^2)}{\eta}, \tag{2.7}$$

and that the correlation of the rainfall process satisfies

$$\mathrm{corr}(\tilde{Y}_{\mathrm{A}}(t), \tilde{Y}_{\mathrm{B}}(t+h)) = \frac{\lambda''(l)}{\lambda'} \mathrm{e}^{-\eta h}. \tag{2.8}$$

Thus, the correlation has a simple product form, factorizing into separate space and time components. When the rain cell radii are exponentially distributed, $\lambda''(l)$ can be approximated as in (2.3). In this case, the space–time correlation structure of the rainfall intensity is

$$\mathrm{corr}(\tilde{Y}_{\mathrm{A}}(t), \tilde{Y}_{\mathrm{B}}(t+h)) \simeq \left(1 + \frac{\rho l}{4}\right) \mathrm{e}^{-\rho l/2} \, \mathrm{e}^{-\eta h}. \tag{2.9}$$

Figure 1 shows examples of the approximate correlation of the rainfall intensity (continuous lines) as given by (2.9), for a particular set of parameter values, and for a range of spatial (l) and temporal (h) lags. The corresponding exact correlation function (dotted lines) is given by (2.8), where numerical integration is used to determine $\lambda''(l)$ from (2.1). The pairs of curves are reasonably close for spatial lags up to a few kilometres, or for time lags of a day or more. If required, better approximations can be obtained by choosing a slightly different value for the constant k; the simple expressions for $\lambda''(l)$ in this paper require $k = \pi/2$.

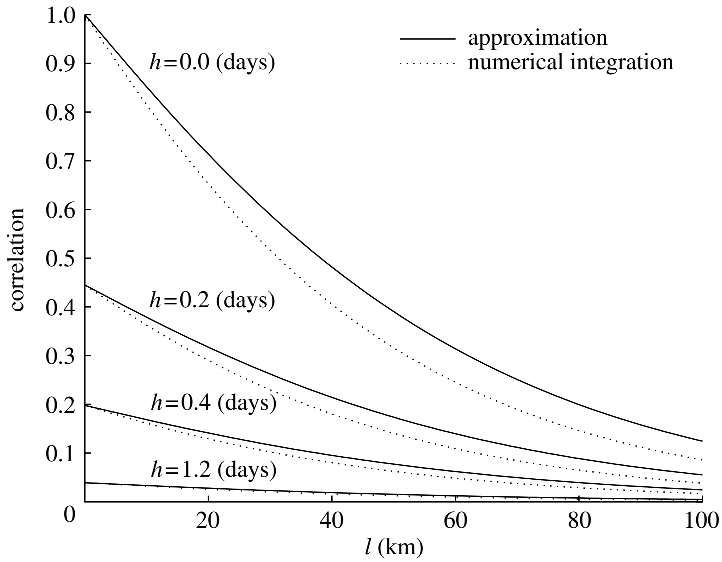

Figure 1. Exact correlation (given by (2.8)) and approximate correlation (given by (2.9)) of the rainfall intensity as functions of distance l and time lag h; the rain cell duration and radius are each assumed to be exponentially distributed, with parameters $\mu_D = \eta^{-1} = 0.25$ days and $\mu_W = \rho^{-1} = 16.6$ km.

3. Soil moisture dynamics

(a) *Simplifying assumptions*

We consider vertically averaged soil moisture content interpreted at the daily time-scale. We concentrate on the spatial soil moisture dynamics induced by spatial variability in rainfall, and assume other sources of variability to be negligible. Thus, the soil moisture dynamics at a single site may be described by a simplified water balance equation,

$$nZ_r \frac{\mathrm{d}S(t)}{\mathrm{d}t} = I(t) - E(s) - L(s), \tag{3.1}$$

where, on the left-hand side of the equation, the dimensionless random variable $S(t)$ represents the relative soil moisture level, n is the soil porosity and Z_r is the depth of the root zone. On the right-hand side, the terms $I(t)$, $E(s)$ and $L(s)$ are, respectively, the rates of infiltration, evapotranspiration and leakage; the last two rates depend on the current level, s, of soil moisture. The infiltration is determined by the random process of precipitation, while evapotranspiration and leakage are deterministic functions. Infiltration and evapotranspiration are dependent on the vegetation at the site, while leakage depends on the soil properties.

It is convenient to standardize the rates E and L, and to write

$$\xi(s) = \frac{E(s)}{nZ_r} + \frac{L(s)}{nZ_r}.$$

In general, the loss function, ξ, can be approximated conveniently by a piecewise linear function as described, for example, by Rodríguez-Iturbe *et al.* (1999). However, here we use the simpler linear form $\xi(s) = as$, where the parameter a will depend on vegetation and soil characteristics. The loss function may be also defined in a non-standardized form as Vs, where V is the water loss coefficient expressed in millimetre per day given by the sum of evapotranspiration and leakage ($V = anZ_\mathrm{r}$). A linear loss function means that in dry periods the soil moisture level decays exponentially. It is implicit in the linear form of ξ that runoff losses are assumed to be a negligible component of the average water balance and not a function of soil moisture level. For example, this approximation is reasonable for arid and semi-arid regions where saturation and well-watered conditions are relatively rare. In this case, the vegetation is frequently under water stress and evapotranspiration is well approximated by a linear dependence on soil moisture content (Laio *et al.* 2001; Porporato *et al.* 2001; Porporato *et al.* 2004). An illustration of this situation, for an arid region with sandy soil and where the vegetation is a mixture of trees and grasses, is discussed in Rodríguez-Iturbe *et al.* (1999).

Although relative soil moisture is bounded between 0 and 1, the upper bound is effectively irrelevant in the physical situation described above, and will be ignored in this paper. This approximation means that the infiltration term I need not to be truncated at a value dependent on the current level of soil moisture, and can be taken to be independent of s.

When the rainfall inputs are taken to be a sequence of instantaneous pulses (model M_1), results can be obtained assuming an arbitrary relationship between rainfall depth, Y, and standardized infiltration, say Y^*, at a particular location, allowing for canopy interception. In particular, two simple schemes can be used. One (proportional interception) assumes interception to be proportional to rainfall so that the standardized infiltration is $Y^* = (1-\phi)Y/(nZ_\mathrm{r}) = bY$, where ϕ is the fraction of rainfall that is intercepted. The second scheme (threshold interception) assumes that all rainfall depths lower than a given threshold θ are intercepted and that, above that a fraction c_0 is further intercepted. Thus, $Y^* = c(Y-\theta)$ when $Y > \theta$ and $Y^* = 0$ otherwise, where $c = (1-c_0)/(nZ_\mathrm{r})$ (see Rodríguez-Iturbe *et al.* 1999).

For model M_2, when the rainfall events have a temporal duration, it is convenient to assume a scheme with proportional interception similar to that described above for M_1. Thus, the standardized infiltration, say $\tilde{Y}^*$, is proportional to the rainfall intensity $\tilde{Y}$ (i.e. $\tilde{Y}^* = b\tilde{Y}$). This assumption becomes more and more reasonable with increasing rainfall depth (e.g. for depths greater than 5–10 mm, Kittredge 1948). Clearly, the parameters ϕ, θ and c_0 depend on the plant species and condition of the vegetation.

These approximations afford considerable mathematical simplifications, and enable explicit expressions to be obtained, with consequent gains in understanding of the processes involved.

(*b*) *Soil moisture dynamics for instantaneous rainfall*

We consider first the case when rain cells occur at the space–time points of a Poisson process of rate λ and the rain depths happen instantaneously, as described above (model M_1). In this case, the process of rain events at a fixed site is a temporal Poisson process of rate λ'.

Let $\{T_k\}$ denote the times of the rain events, with corresponding depths Y_k and standardized infiltrations Y_k^* as described above. For the instantaneous rainfall case, the soil moisture level at time t can be represented as a simple sum over all the events that occur during $(0,t]$, together with a contribution from the initial state $S(0)$:

$$S(t) = S(0)\mathrm{e}^{-at} + \sum_{k:\, T_k \le t} Y_k^* \,\mathrm{e}^{-a(t-T_k)}. \tag{3.2}$$

From this representation, it is straightforward to use the properties of the Poisson process to determine the moment generating function $G_S(v;t) = \mathrm{E}(e^{-vS(t)})$ of $S(t)$ in terms of the corresponding moment generating function $G_{Y^*}(v)$ of the standardized infiltration Y^*, and hence to determine properties of the marginal distribution of $S(v;t)$. In particular, for the transient case

$$G_S(v;\, t) = \exp\left\{-vS(0)\mathrm{e}^{-at} - \lambda' \int_0^t [1 - G_{Y^*}(v\,\mathrm{e}^{-au})]\mathrm{d}u\right\}, \tag{3.3}$$

where the form of (3.3) is to be expected given the compound Poisson structure of the rainfall input together with the exponential decay of the soil moisture. It follows immediately that the mean and variance of soil moisture are given by

$$\mathrm{E}(S(t)) = S(0)\mathrm{e}^{-at} + \frac{\lambda'}{a}(1 - \mathrm{e}^{-at})E(Y^*),$$

$$\mathrm{var}(S(t)) = \frac{\lambda'}{2a}(1 - \mathrm{e}^{-2at})\mathrm{E}(\{Y^*\}^2).$$

Corresponding results for the process in equilibrium are simply obtained in the limit as $t \to \infty$.

When the rain depth, Y, is exponentially distributed with parameter $\gamma = 1/\mu_Y$, and the standardized infiltration is proportional to the rain depth $(Y^* = bY)$, then $G_{Y^*}(v) = \gamma/(\gamma + vb)$. In this case, it is straightforward to perform the integration in (3.3), to obtain the generating function in closed form:

$$G_S(v;\, t) = \exp\{-vS(0)\mathrm{e}^{-at}\}\left(\frac{\gamma + vb\mathrm{e}^{-at}}{\gamma + vb}\right)^{\lambda'/a}.$$

In particular, it follows that, in equilibrium, S has a gamma distribution with mean $\lambda' b/(a\gamma)$ and variance $\lambda' b^2/(a\gamma^2)$. We note that it is known (e.g. Rodríguez-Iturbe & Porporato 2004, p. 210) that daily soil moisture data can be well-fitted by a gamma distribution, particularly for arid environments. Further discussion of these model properties and a comparison with the corresponding results obtained for model variant M_2 are postponed to §§3*b* and 4.

For the alternative scheme of threshold interception, when Y is exponentially distributed, $G_{Y^*}(v) = [\gamma + cv(1 - \mathrm{e}^{-\gamma\theta})]/(\gamma + cv)$, so that the equilibrium mean and variance of $S(t)$ are, respectively,

$$\mathrm{E}(S(t)) = \frac{c\lambda'\,\mathrm{e}^{-\gamma\theta}}{a\gamma}$$

and

$$\mathrm{var}(S(t)) = \frac{c^2\lambda'\,\mathrm{e}^{-\gamma\theta}}{a\gamma^2}.$$

The joint distribution of soil moisture, $S_{\mathrm{A}}(t)$ and $S_{\mathrm{B}}(t+h)$, at two sites, say A and B, that are l apart can be deduced similarly by representing each variable by a sum of independent terms, each of which is a sum over the events of a Poisson process. In particular, $S_{\mathrm{A}}(t)$ can be separated into two terms: a sum over events before t that affect both A and B and occur in a Poisson process of rate $\lambda''(l)$, and a sum over rain events before t that only affect A and occur in a Poisson process of rate $\lambda'-\lambda''(l)$. The two Poisson processes here are independent. Similarly, it is convenient to represent $S_{\mathrm{B}}(t+h)$ as a sum of three terms: a sum over events before t that affect both A and B (a Poisson process of rate $\lambda'(l)$), a sum over events before t that only affect B (a Poisson process of rate $\lambda'-\lambda''(l)$), and a sum over events in $(t,t+h)$ that affect B (and may or may not also affect A, and occur in a Poisson process of rate λ'). The dependence between $S_{\mathrm{A}}(t)$ and $S_{\mathrm{B}}(t+h)$ is determined by the first term in each of these two decompositions, for which the Poisson process is in common; all other pairs of Poisson processes are independent.

Using the properties of these Poisson processes, the joint moment generating function of $S_{\mathrm{A}}(t)$ and $S_{\mathrm{B}}(t+h)$ can be found. For simplicity, we give here only the expression for the case of proportional interception:

$$\begin{aligned}
&\mathrm{E}(\mathrm{e}^{-v_{\mathrm{A}}S_{\mathrm{A}}(t)-v_{\mathrm{B}}S_{\mathrm{B}}(t+h)})\\
&\quad=\exp\Bigg\{-v_{\mathrm{A}}S_{\mathrm{A}}(0)\mathrm{e}^{-a_{\mathrm{A}}t}-v_{\mathrm{B}}S_{\mathrm{B}}(0)\mathrm{e}^{-a_{\mathrm{B}}(t+h)}-\lambda'\int_0^h[1-G_Y(v_{\mathrm{B}}b_{\mathrm{B}}\mathrm{e}^{-a_{\mathrm{B}}u})]\mathrm{d}u\\
&\qquad-\lambda''(l)\int_0^t[1-G_Y(v_{\mathrm{A}}b_{\mathrm{A}}\,\mathrm{e}^{-a_{\mathrm{A}}u}+v_{\mathrm{B}}b_{\mathrm{B}}\,\mathrm{e}^{-a_{\mathrm{B}}(u+h)})]\mathrm{d}u\\
&\qquad-(\lambda'-\lambda''(l))\int_0^t[2-G_Y(v_{\mathrm{A}}b_{\mathrm{A}}\,\mathrm{e}^{-a_{\mathrm{A}}u})-G_Y(v_{\mathrm{B}}b_{\mathrm{B}}\,\mathrm{e}^{-a_{\mathrm{B}}(u+h)})]\mathrm{d}u\Bigg\}.
\end{aligned}$$

Note that this expression allows the vegetation and soil characteristics, and hence the parameters a and b, to differ at the two sites; further consideration of this physical situation is deferred to Rodríguez-Iturbe *et al.* (submitted).

In particular, for the process in equilibrium, the covariance between $S_{\mathrm{A}}(t)$ and $S_{\mathrm{B}}(t+h)$ is

$$c_{\mathrm{AB}}(h)=\mathrm{cov}(S_{\mathrm{A}}(t),S_{\mathrm{B}}(t+h))=\lambda''(l)\frac{\mathrm{e}^{-a_{\mathrm{B}}h}}{a_{\mathrm{A}}+a_{\mathrm{B}}}b_{\mathrm{A}}b_{\mathrm{B}}\mathrm{E}(Y^2), \tag{3.4}$$

so that the corresponding correlation is

$$\mathrm{corr}(S_{\mathrm{A}}(t),S_{\mathrm{B}}(t+h))=\frac{\lambda''(l)}{\lambda'}\frac{2\sqrt{(a_{\mathrm{A}}a_{\mathrm{B}})}}{a_{\mathrm{A}}+a_{\mathrm{B}}}\mathrm{e}^{-a_{\mathrm{B}}h}. \tag{3.5}$$

Thus, the correlation of soil moisture factorizes into spatial and temporal components, this factorization being an immediate consequence of that for the rainfall process given in (2.8).

For the general interception case, the covariance is given by

$$c_{\mathrm{AB}}(h)=\lambda''(l)\frac{\mathrm{e}^{-a_{\mathrm{B}}h}}{a_{\mathrm{A}}+a_{\mathrm{B}}}\mathrm{E}(Y^{\mathrm{A}*}Y^{\mathrm{B}*}), \tag{3.6}$$

where $Y^{\mathrm{A}*}$ and $Y^{\mathrm{B}*}$ denote, respectively, the standardized infiltrations at A and B, corresponding to *the same* rain depth Y. In the homogeneous case ($a_{\mathrm{A}}=a_{\mathrm{B}}=a$), the correlation reduces to $\lambda''(l)\mathrm{e}^{-ah}/\lambda'$, which applies regardless of the form of interception assumed. In the non-homogeneous case, when A and B have different parameters, it is interesting to compare the effects of proportional and threshold interception on the correlation of soil moisture at the two sites. It can be shown that these correlations are in fact the same, to first order in $\gamma\theta$.

(*c*) *Soil moisture dynamics for temporally distributed rainfall*

We now turn to the case when rain cells have a temporal duration (model variant M_2). With the assumptions described above, of linear losses and proportional interception, the differential equation for the relative soil moisture (3.1) can be rewritten as

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = -aS(t) + b\tilde{Y}(t), \tag{3.7}$$

where $\tilde{Y}$ denotes the total rainfall intensity, as specified in (2.4) but with the spatial location $\boldsymbol{u}$ suppressed.

The solution of (3.7) gives the dynamics of the soil moisture process as a function of rainfall. Thus, in equilibrium

$$S(t) = \int_0^\infty \mathrm{e}^{-a\omega}\, b\tilde{Y}(t-\omega)\mathrm{d}\omega, \tag{3.8}$$

from which, together with (2.5) it follows immediately that the expected equilibrium value of the relative soil moisture is

$$\mathrm{E}(S(t)) = \frac{\lambda'\mu_D\mu_X b}{a}.$$

When the cell radii and durations are exponentially distributed with means ρ^{-1} and η^{-1}, respectively, $\mathrm{E}(S(t))=2\pi\lambda\mu_X b/(\rho^2\eta a)$. Note that the integral form for $S(t)$ in (3.8) is directly analogous to the sum (3.2) that applies to model variant M_1, where the rainfall input is in discrete time.

With proportional interception, a comparison of the soil moisture means for the two models shows that, as is to be expected, both are proportional to the mean rainfall intensity ($\lambda'\mu_D\mu_X$ in model M_2 and $\lambda'\mu_Y$ in model M_1) with a constant of proportionality b/a. Similarly, for threshold interception, the mean for model M_1 has the same form but with b replaced by $c\,\mathrm{e}^{-\gamma\theta}$. As described above, both the loss function, $\xi(s)$, and the infiltrations Y^* and $\tilde{Y}^*$ have been standardized by the factor nZ_r, so that the ratios b/a and c/a are independent of nZ_r. Thus, in each case, the mean relative soil moisture does not depend on the soil characteristics encapsulated in the product nZ_r. This arises because the model does not allow for the upper bound in soil moisture due to the saturation of the soil column. As noted before, the effect of this simplification is not of concern so long as well-watered conditions are relatively rare. Figure 2 shows a comparison of mean soil moisture between the present model (variant M_1) which has no saturation-excess runoff, and a similar nonlinear model previously studied by the authors (e.g. Porporato *et al.* 2004) which has instantaneous rainfall pulses and comparable linear losses, but where $S(t)$ is bounded above by one. The figure shows that for water limited ecosystems (e.g. $\lambda'_R < 0.3$) the means of the two processes are very similar.

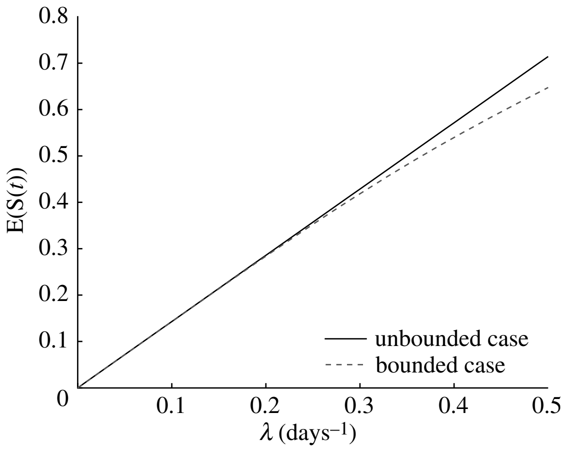

Figure 2. Comparison between the mean relative soil moisture obtained with the present model (variant M_1), where soil moisture is unbounded and a similar model that includes the bound at $s=1$ (e.g. Porporato *et al.* 2004), as a function of the parameter λ'. The parameters in this example are $\gamma=0.10\ \mathrm{mm}^{-1}$, $a=0.070\ \mathrm{day}^{-1}$ and $b=0.010\ \mathrm{mm}^{-1}$.

Now consider two locations, A and B, a distance l apart, with relative soil moistures S_{A} and S_{B}, each of which can be expressed as an integral of the form (2.5) but with parameters a and b, and rainfall intensity $\tilde{Y}$, depending on the respective locations. Thus, the equilibrium covariance, $c_{\mathrm{AB}}(h)$, between $S_{\mathrm{A}}(t)$ and $S_{\mathrm{B}}(t+h)$ can be written in terms of the covariance of the rainfall process as

$$c_{\mathrm{AB}}(h) = \int_0^\infty \int_0^\infty b_{\mathrm{A}} b_{\mathrm{B}}\, \mathrm{e}^{-a_{\mathrm{A}}\omega - a_{\mathrm{B}}\tau} \mathrm{cov}(\tilde{Y}_{\mathrm{A}}(t-\omega), \tilde{Y}_{\mathrm{B}}(t+h-\tau))\mathrm{d}\omega\, \mathrm{d}\tau.$$

We assume that the cell durations are exponentially distributed with parameter η. It is then straightforward to use equation (2.6) to show that, for $h \geq 0$

$$c_{\mathrm{AB}}(h) = \lambda''(l) b_{\mathrm{A}} b_{\mathrm{B}} \mathrm{E}(X^2)\left\{\frac{2\,\mathrm{e}^{-a_{\mathrm{B}}h}}{(a_{\mathrm{A}}+a_{\mathrm{B}})(\eta^2 - a_{\mathrm{B}}^2)} - \frac{\mathrm{e}^{-\eta h}}{\eta(\eta - a_{\mathrm{B}})(\eta + a_{\mathrm{A}})}\right\}, \tag{3.9}$$

where it is important to note above that the suffix B corresponds to the location at the later time. The approximate expression (2.3) for $\lambda''(l)$ can be substituted if required. The corresponding covariance for model M_1, given in equation (3.4), can be obtained from (3.9) by taking a limit as $\eta \to \infty$ with XD replaced by Y (so that $\mathrm{E}(X)/\eta = \mathrm{E}(Y)$ and $2\mathrm{E}(X^2)/\eta^2 = \mathrm{E}(Y^2)$).

Letting $l \to 0$ and $h \to 0$ in (3.9), so that the locations A and B coincide and $\lambda''(l) \to \lambda'$, it follows that the variance, $\sigma_S^2 = \mathrm{var}[S_{\mathrm{A}}(t)]$, of the soil moisture process is

$$\sigma_S^2 = \frac{\lambda' b_{\mathrm{A}}^2 \mathrm{E}(X^2)}{a_{\mathrm{A}} \eta(\eta + a_{\mathrm{A}})}, \tag{3.10}$$

and hence that the spatial-temporal correlation of the soil moisture process is given by

$$\operatorname{corr}(S_A(t), S_B(t+h)) = \frac{\lambda''(l)}{\lambda'}\sqrt{\{a_A a_B(\eta + a_A)(\eta + a_B)\}}\left\{\frac{2\eta\,\mathrm{e}^{-a_B h}}{(a_A + a_B)(\eta^2 - a_B^2)} - \frac{\mathrm{e}^{-\eta h}}{(\eta - a_B)(\eta + a_A)}\right\}. \tag{3.11}$$

Note the factorization of the correlation into spatial and temporal components, and the similarity of (3.11) to the expression (3.5) obtained with the instantaneous rainfall model M_1, although of course the former includes a decay term on the temporal scale of the rain cell durations as well as one corresponding to that of the loss function.

4. Spatial-temporal variability of soil moisture with uniform vegetation

(*a*) *The spatial-temporal correlation function*

In §3, properties of the joint distribution of soil moisture at two locations and at two distinct times were obtained, allowing for the dependence resulting from that of the input process of rainfall. The results were obtained conditionally upon the vegetation and soil characteristics at those locations, which were not assumed to be the same. In this section, we will examine in detail the consequences of these results in the simplest case that of uniform soil and vegetation, focusing on the model variant M_2. As described earlier, results for M_1 can be obtained by considering the limiting case as $\eta \to \infty$ with Y replacing XD. Further examination of the heterogeneous case is deferred to Rodríguez-Iturbe *et al.* (submitted).

With spatially uniform soil and vegetation, so that $a_A = a_B = a$ and $b_A = b_B = b$, equations (3.9)–(3.11) for the covariance, variance and correlation of soil moisture simplify to become, respectively,

$$c_{AB}(h) = \lambda''(l) b^2 \mathrm{E}(X^2)\left\{\frac{\eta\,\mathrm{e}^{-ah} - a\,\mathrm{e}^{-\eta h}}{a\eta(\eta^2 - a^2)}\right\}, \tag{4.1}$$

$$\sigma_S^2 = \frac{\lambda' b^2 \mathrm{E}(X^2)}{a\eta(\eta + a)} = \frac{b^2}{a(\eta + a)}\sigma_{\tilde{Y}}^2, \tag{4.2}$$

and

$$\operatorname{corr}(S_A(t), S_B(t+h)) = \frac{\lambda''(l)\{\eta\,\mathrm{e}^{-ah} - a\,\mathrm{e}^{-\eta h}\}}{\lambda'(\eta - a)} = \frac{\lambda''(l)}{\lambda'}(1 + o(h)), \tag{4.3}$$

as $h \to 0$. Thus, for small time lags, the spatial-temporal correlation function is approximately just $\lambda''(l)/\lambda'$.

Note that the variance of the relative soil moisture is directly proportional to that of the rainfall process. Since $\eta \gg a$, the constant of proportionality is approximately $b^2/(a\eta)$. As both a and b are inversely proportional to the product nZ_r, it follows that for a fixed soil porosity n, the variance σ_S^2 is approximately

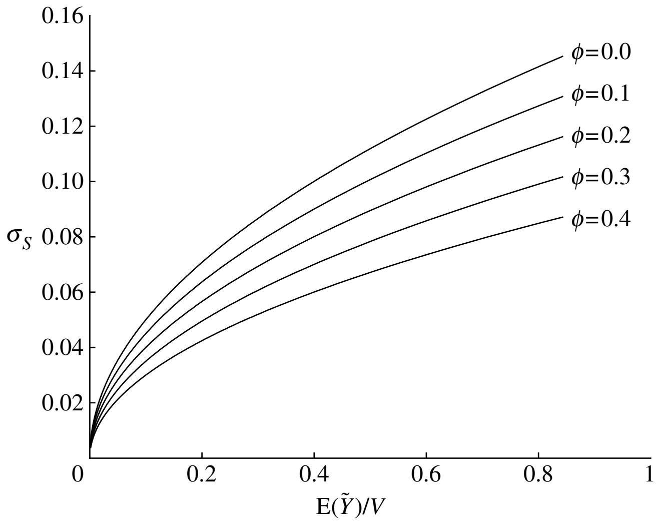

Figure 3. Standard deviation of the relative soil moisture versus the ratio $\mathrm{E}(\tilde{Y})/V$ for different values of the parameter ϕ. The graph depends on the parameters $\mu_D=\eta^{-1}=0.25$ days, $\mu_X=25$ mm day^{-1}, $V=anZ_\mathrm{r}=7.0$ mm day^{-1} and $nZ_\mathrm{r}=500$ mm.

inversely proportional to the root depth, Z_r. In comparison, for model M_1, the constant of proportionality is b^2/a for proportional interception and $c^2\,\mathrm{e}^{-\gamma\theta}/a$ for threshold interception, where c is also inversely proportional to nZ_r, and thus similar conclusions apply. These results are dependent on the assumption that the upper bound on soil moisture can be ignored. When the effective soil layer is very shallow and the soil is often saturated, the above approximation is not reliable. In fact, when the soil moisture is bounded at $s=1$, which is a relatively infrequent situation in water-limited ecosystems, the variance is necessarily reduced (see, e.g. figure 6 in Rodríguez-Iturbe *et al.* 1999).

Figure 3 shows the standard deviation of the relative soil moisture as a function of the ratio $\mathrm{E}(\tilde{Y})/V$ for different values of the rainfall interception coefficient $\phi=1-bnZ_\mathrm{r}$, when the cell intensities (X) are exponentially distributed. Here, $\mathrm{E}(\tilde{Y})/V$ represents the ratio of the expected rainfall intensity to the potential soil water loss, and is comparable with the aridity index, R, proposed by the UNEP (1992, p. 69). According to this index a region may be classified as hyperarid $(R<0.05)$, arid $(0.05<R<0.20)$, semi-arid $(0.20<R<0.50)$, dry subhumid $(0.50<R<0.65)$ and humid $(R>0.65)$. Thus, figure 3 shows the standard deviation of the relative soil moisture moving from arid towards humid climates; note however that in humid conditions the variance is probably overestimated because of the presence of runoff.

The spatial correlation of the relative soil moisture is determined entirely by that of the rainfall process (equation 2.8) and thus has a similar form. On the other hand, the temporal autocorrelation of the soil moisture is governed not only by the temporal durations of the rain cells (mean $1/\eta$) but also by the rate (a) at which soil moisture decays. In general, the parameter a is of the order of 10^{-2} day^{-1}, while η assumes higher values ($\sim 10^{-1}$ day^{-1}). Given the difference of one order of magnitude between the two parameters, the autocorrelation of the soil moisture is mostly controlled by the soil water losses (parameter a) rather

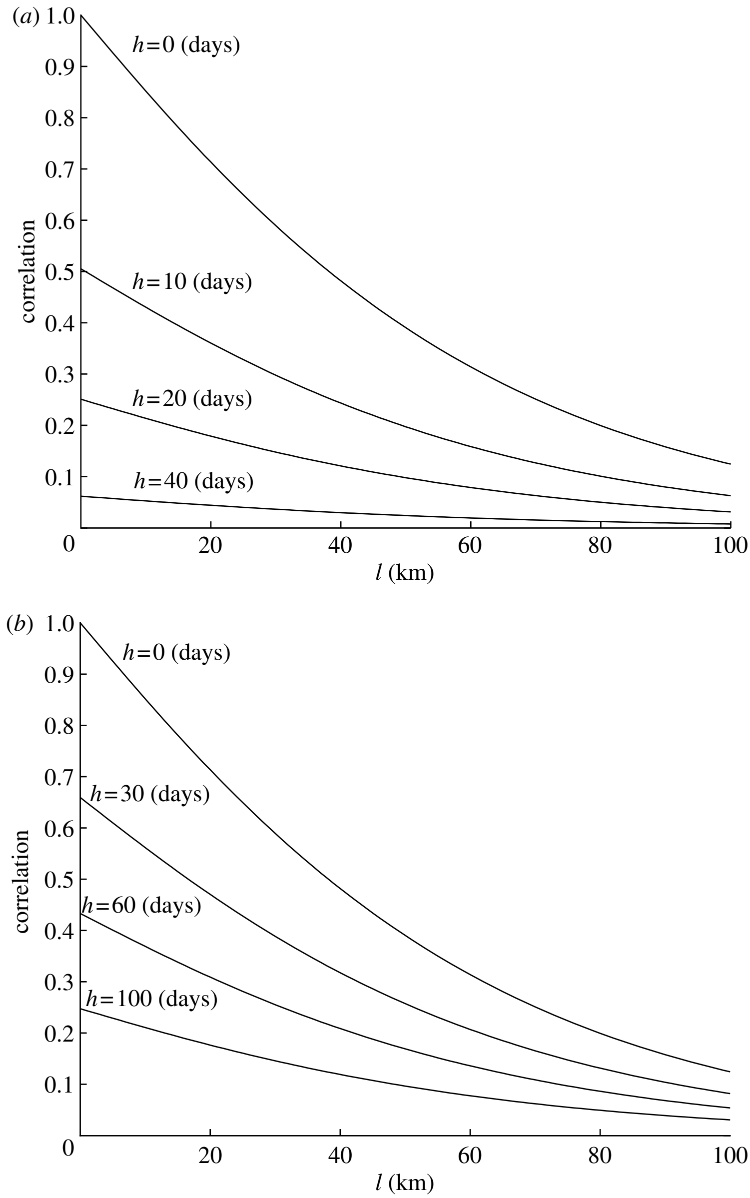

Figure 4. Examples of correlation of the relative soil moisture, with $nZ_r = 100$ mm (a) and $nZ_r = 500$ mm (b), as a function of distance l and for assigned time lags h. Parameters of the rainfall model are as for figure 1, with $V = 7.0$ mm day^{-1}.

than by the temporal dynamics of rainfall (parameter η). As the soil becomes shallower the parameter a increases and the temporal dynamics of the rainfall may play a more significant role in the temporal correlation of the soil moisture. A further paper studies the space–time correlation of the soil moisture for the case of a heterogeneous vegetation field (Rodríguez-Iturbe *et al.* submitted).

Two examples of the correlation of relative soil moisture (computed using the approximation to $\lambda''(l)$ given in (2.3)) are shown in figure 4. The parameter

Table 1. *Estimated values of the parameter Λ of the Gaussian approximation to the spatial autocorrelation as a function of ρ (ρ^{-1} is the mean rain cell radius)*

ρ (km^{-1})	0.01	0.02	0.04	0.06	0.08	0.10	0.20	0.40	0.60
Λ (km^{-1})	0.0042	0.0079	0.015	0.023	0.030	0.037	0.075	0.15	0.22

values are the same in both figures, except that nZ_r takes the value 100 mm in figure 4*a* and 500 mm in figure 4*b*. The increase in the root depth leads to a consistent rise in the correlation in time of the relative soil moisture. The deeper the soil, the less sensitive its saturation becomes to fluctuations induced by rainfall, evapotranspiration and leakage.

(*b*) *Relative soil moisture instantaneous in time and averaged in space*

The effect of scale on the description of soil moisture dynamics is a crucial problem in hydrology. Several meteorological and hydrological processes related to land–atmosphere interaction and flood formation, as well as eco-hydrological patterns (water and CO_2 fluxes, biomass allocation, etc.) are impacted by the soil moisture distributions at large spatial scales. Moreover, the soil moisture variability within a given area may affect results from hydrological models. Such models use a grid schematization of the surface in which the soil moisture is assumed uniform in each cell of the grid. The extensive use of remote sensing images makes the problem even more critical for a correct interpretation of what is being averaged over different spatial scales.

The variance, $\sigma^2_{S_L}$, of the relative soil moisture process averaged over a given square area of side L can be obtained by integrating the covariance of the soil moisture process in space (e.g. VanMarcke 1983, p. 382):

$$\sigma^2_{S_L} = \frac{1}{L^4}\int_{L\times L}\int_{L\times L} \mathrm{cov}(S(\boldsymbol{w},t),S(\boldsymbol{v},t))\mathrm{d}\boldsymbol{w}\,\mathrm{d}\boldsymbol{v}$$
$$= \frac{4}{L^4}\int_0^L\int_0^L (L-u_1)(L-u_2)\mathrm{cov}(S(\boldsymbol{w},t),S(\boldsymbol{w}+\boldsymbol{u},t))\mathrm{d}u_1\,\mathrm{d}u_2, \qquad (4.4)$$

where the previous notation is extended so that $S(\boldsymbol{w}, t)$ denotes the soil moisture at location $\boldsymbol{w}$ at time t.

For homogeneous vegetation, the integral of (4.4) can be found explicitly by using a Gaussian function of the form $2\pi\lambda\rho^{-2}\exp\{-(\Lambda l)^2\}$ to approximate $\lambda''(l)$, rather than using the exact form determined by (2.1). The parameter Λ is set to give a good fit to this function. Table 1 provides estimates of Λ for a wide range of values of ρ. It can be seen that Λ is essentially linear in ρ.

With such an assumption, it is possible to derive the variance of the instantaneous soil saturation process averaged over a square area of size $L\times L$:

$$\sigma^2_{S_L} \simeq \frac{2\pi\lambda \mathrm{E}(X^2)b^2}{a\eta(\eta+a)\rho^2(\Lambda L)^4}(\sqrt{\pi}L\Lambda\mathrm{erf}(L\Lambda) + \mathrm{e}^{-(\Lambda L)^2} - 1)^2. \qquad (4.5)$$

The above approximation can be compared with the exact variance of the averaged process computed via numerical integration. Figure 5*a*,*b* shows excellent agreement of the functions at small and medium scales. The agreement is not so

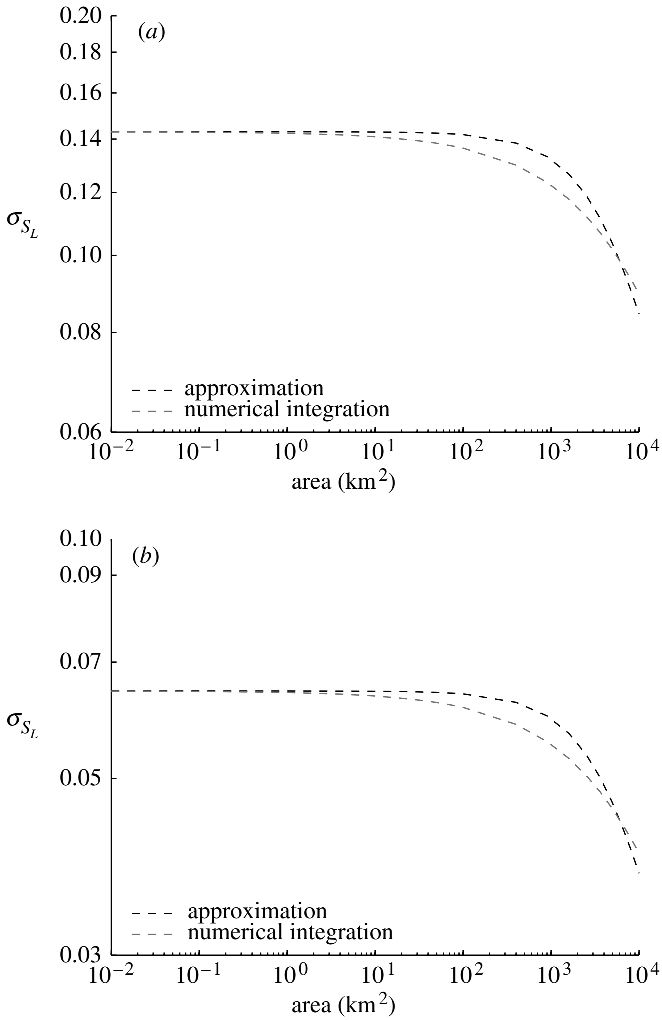

Figure 5. Comparison of the approximate standard deviation for spatially averaged relative soil moisture obtained using the Gaussian form for the spatial correlation function, with that computed via numerical integration using (2.3). Parameters of the model are based on analysis of empirical Italian data (Rodríguez-Iturbe *et al.* submitted): $\lambda=1.66\times10^{-4}$ day^{-1} km^{-2}, $\mu_D=\eta^{-1}=0.25$ days, $\mu_X=25.4$ mm day^{-1}, $\mu_W=\rho^{-1}=16.6$ km, $\Lambda=0.022$ km^{-1}, $V=7.0$ mm day^{-1} and $\phi=0.20$. In (*a*) $nZ_r=100$ mm and in (*b*) $nZ_r=500$ mm.

good for averaging areas larger than 100 km^2, but the approximation can be considered reasonable at all scales. Although the comparison is dependent on the set of parameters being used, it was nevertheless satisfactory for all values used in the examples of this paper. The approximation has been used in the preparation of the subsequent figures in this paper showing properties involving spatial averaging.

Figure 6 shows the effects of spatial averaging on the ratio between the standard deviation of the averaged process and the point standard deviation. This ratio depends only on the parameter ρ and the spatial scale L, as is easily verified by comparing equations (3.10) and (4.5) (recall that the spatial correlation parameter Λ of relative soil moisture depends only on ρ). This

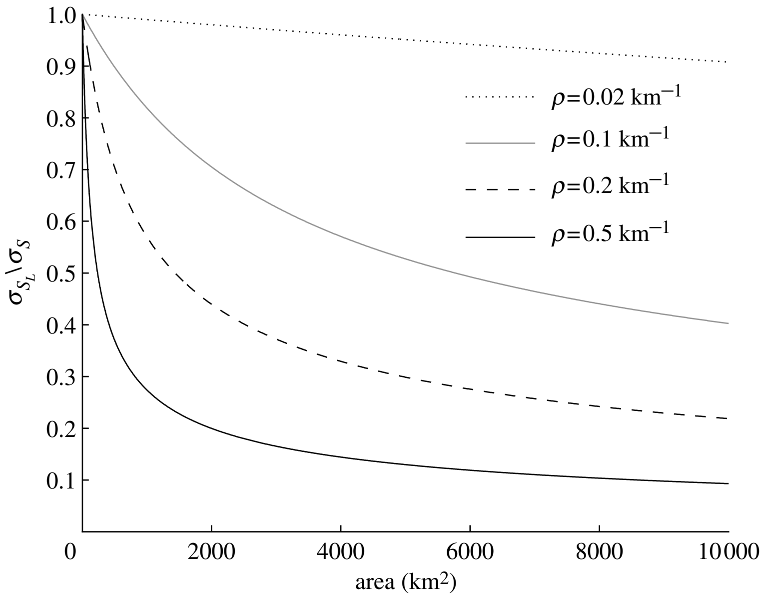

Figure 6. Ratio between the standard deviation (σ_{S_L}) of the spatially averaged relative soil moisture process and the point standard deviation (σ_S), for different values of ρ, ranging from 0.02 to 0.5 km^{-1}.

means that it is representative of any soil and climate condition, and depends only on the characteristic scale of the rain cells. The mean size of rain cells controls the rate of decay of the standard deviation with increasing spatial scale, with the ratio decaying much more rapidly for large rain cells (small ρ).

(*c*) *Relative soil moisture averaged in space and time*

It is important to study the properties of the relative soil moisture process integrated also over time. Their characterization may help in the parameterization of numerical models and in the interpretation of data sampled at coarse space–time resolution.

The variance, $\sigma^2_{S_L^T}$, of the process averaged over a square spatial region of side L and a temporal interval of length T is

$$\sigma^2_{S_L^T} = \frac{1}{L^4T^2}\int_0^T\int_0^T\int_{L\times L}\int_{L\times L}\mathrm{cov}(S(\boldsymbol{w},\tau_1),S(\boldsymbol{u},\tau_2))\mathrm{d}\boldsymbol{w}\,\mathrm{d}\boldsymbol{u}\,\mathrm{d}\tau_1\,\mathrm{d}\tau_2. \tag{4.6}$$

Using equations (3.9) and (4.6) together with the Gaussian approximation to the spatial correlation described in §4*b*, we obtain

$$\sigma^2_{S_L^T} \simeq \frac{2\pi\lambda\mathrm{E}(X^2)}{\eta(\rho T)^2(\Lambda L)^4}\left(\sqrt{\pi}L\Lambda\,\mathrm{erf}(L\Lambda)+\mathrm{e}^{-(\Lambda L)^2}-1\right)^2 \times\frac{2b^2}{a^3\eta^2(\eta^2-a^2)}\left(\eta^3(\mathrm{e}^{-aT}-1+aT)-a^3(\mathrm{e}^{-\eta T}-1+\eta T)\right). \tag{4.7}$$

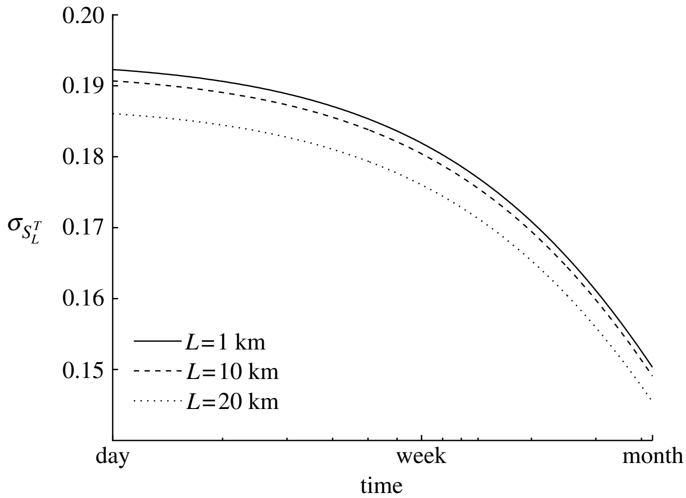

Figure 7. Standard deviation of relative soil moisture averaged over a square spatial area with side $L=1$, 10 and 20 km, and in time over durations ranging from 1 h to one month; $nZ_r = 100$ mm. Remaining parameters of the model are as for figure 5.

Figure 7 illustrates equation (4.7). For the parameters used there and for averaging areas of 1, 100 and 400 km^2, the temporal averaging leads to a negligible reduction in the standard deviation for periods of less than 3 days; the smoothing effect becomes increasingly more pronounced for longer averaging durations.

The standard deviation of the integrated process as a function of the product of soil porosity and depth, nZ_r, is shown in figure 8 for four different temporal averaging windows, and for a region of 1 km^2. The averaging in time produces a departure from the power law corresponding to the instantaneous process, particularly for small values of the effective root depth.

Figure 9 shows a summary of the effects of both temporal and spatial averaging on the ratio of the standard deviation of the averaged relative soil moisture process ($\sigma_{S_L^T}$) to the standard deviation of the instantaneous process (σ_S). For $\eta \gg a$, this ratio is easily seen to be well-approximated by a product of a function of ΛL and a function of Ta. As already noted, Λ is approximately linear in ρ. Thus, the graphs are plotted in terms of dimensionless parameters Ta and ρL and the ratio is, to a good approximation, independent of the values of all other model parameters; $\eta = 4.0$ days^{-1} was used in figure. The two graphs are complementary: figure 9*a* shows the ratio $\sigma_{S_L^T}/\sigma_S$ as a function of space for a set of time-intervals and figure 9*b* shows the same ratio as a function of Ta for different $L\rho$. The dominant roles played by the characteristics of the mean cell radius ρ^{-1} on the spatial scaling and of the soil loss coefficient a on the temporal scaling of the process are clear. In particular, the soil moisture variability of the averaged process starts to decrease only for averaging areas with characteristic size $L\rho$ greater than 1 and loses most of its variability when the spatial scale parameter $L\rho$ approaches the value 100. Averaging in time produces similar results that are related to the normalized water loss coefficient, a, which represents the temporal memory of the system. As a gets larger the system dissipates more rapidly and tends to a faster decrease of its temporal variability.

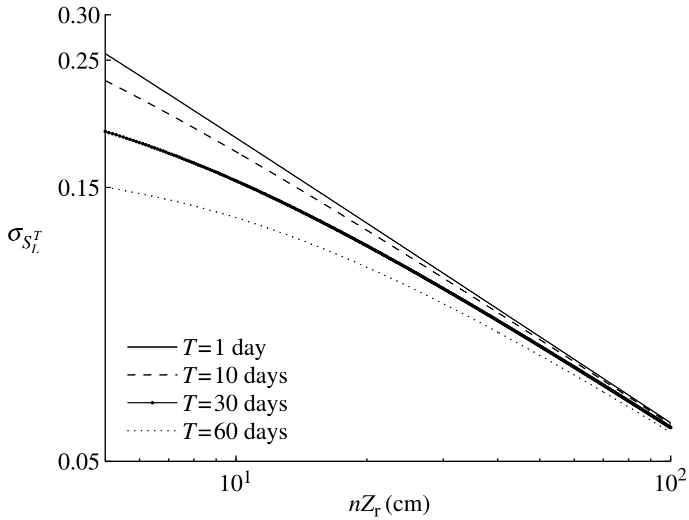

Figure 8. Standard deviation of the relative soil moisture process averaged over a square area of 1 km^2 and over different temporal durations as a function of the product of soil porosity and depth, nZ_r. Remaining parameters of the model are as for figure 5.

5. Final discussion

Soil moisture fluctuations in space and time have considerable relevance in many research areas. Besides the instantaneous process, the probabilistic structure of the spatial, temporal and space–time averaged relative soil moisture is also of key interest in hydrological modelling, remote sensing, eco-hydrology and soil atmosphere interaction.

The soil moisture dynamics have been explored following an approach based on a stochastic differential equation driven by stochastic spatial-temporal rainfall forcing. Two variants for the rainfall model are considered: in one random rainfall depths are deposited instantaneously in a Poisson process in space–time, while in the other the rainfall has a random intensity for a random duration. In both cases, the rainfall has a random spatial extent. Properties of soil moisture for both model variants are qualitatively similar. By modelling soil moisture as a continuous-time random process $\{S(\boldsymbol{w},t)\}$ that represents the instantaneous relative soil moisture state in continuous space and time, it is possible to infer the properties of the processes averaged over a range of spatial and temporal scales. The model allows the analytical derivation of probability distributions and their properties, in particular the first and second order moments and correlation structure of relative soil moisture, exhibiting their dependence on physical parameters such as root depth, water soil loss coefficient, vegetation interception and the mean intensity, duration and spatial extent of rain cells. The model is highly idealized but contains the most important features affecting the probabilistic structure of soil moisture when topography and changes in soil properties are not dominant factors, and its mathematical tractability enables gains in understanding of the physical processes to be achieved.

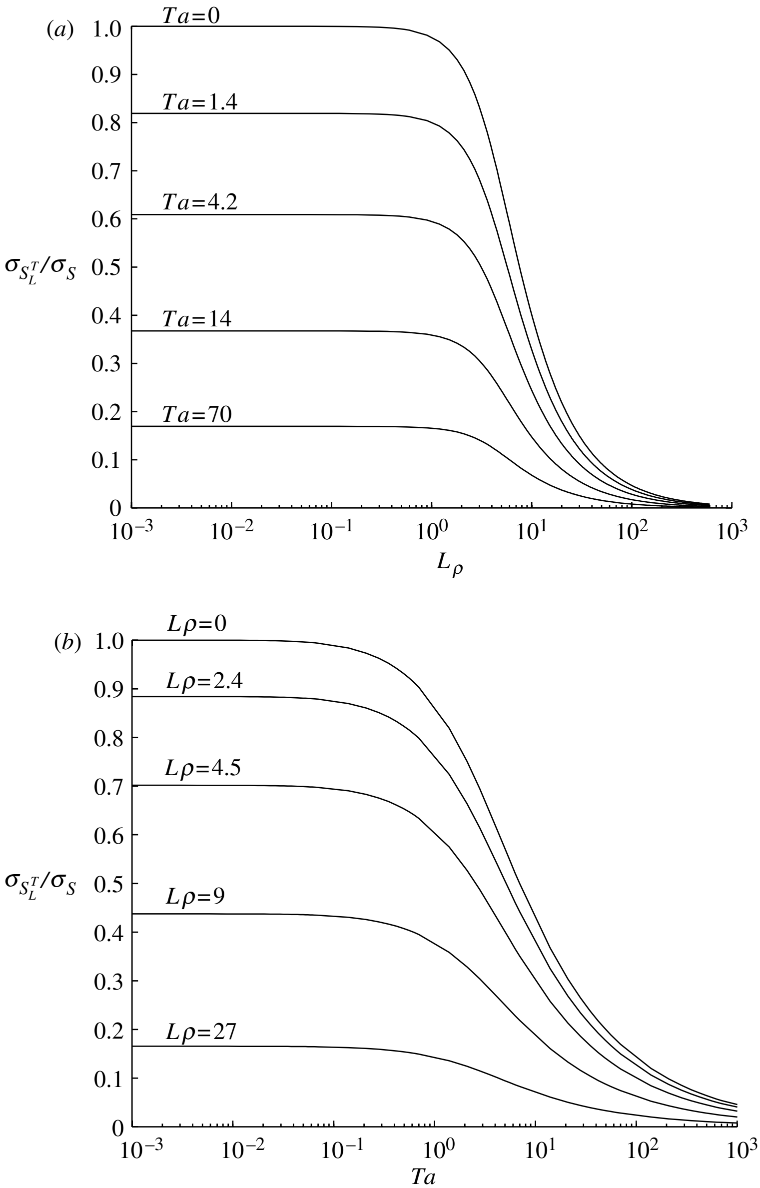

Figure 9. Ratio between the approximate standard deviation of relative soil moisture averaged in time and space ($\sigma_{S_L^T}$), and the point standard deviation (σ_S), as a function of (*a*) space for a set of time scales and (*b*) time for a set of space scales. The graphs describe the general behaviour of the soil moisture averaged in space and time, and are plotted for $\eta = 4.0\ \text{days}^{-1}$.

In water-limited ecosystems, where the upper bound on soil moisture is irrelevant and can be ignored, the variance of relative soil moisture was found to be directly proportional to the variance of the rainfall process, with a constant of proportionality that is approximately inversely related to the soil water capacity (nZ_r) (the product of the soil porosity, n and root depth, Z_r). The spatial correlation of the relative soil moisture reflects that of the rainfall process, while the temporal correlation is mainly controlled by water losses through the dissipative parameter a. The effect is that the temporal correlation of the soil moisture generally increases with effective soil depth.

In the present paper, we have focussed on a specific case in which the vegetation and the soil properties are assumed homogeneous. In this case, expressions for the variance of averaged relative soil moisture have been derived in closed form using a Gaussian approximation for the spatial correlation function of rainfall. Averaging over time periods up to two or three days does not significantly modify the soil moisture variance since the parameter a is, generally, much smaller than one; values of T such that Ta is around one or more are needed for significant reductions in the variance to occur. Similarly, spatial averaging significantly smoothes the relative soil moisture process when the spatial scale L is such that $L\rho$ exceeds one.

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